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<td>9 + Mar 1-5</td>
<td>Introduction to “Working with Dynamic Crop Models”</td>
<td>Wallach et al. 2006 (Course Text)</td>
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<td>• Overview of Book</td>
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Weeks 9 “+” Objectives

1) To have students become familiar with model forms and notation that will be used in the remainder of the course

2) Review basic statistical methods that will be used during the rest of the semester
Dealing with Complexities in Cropping Systems

• What is the purpose of the model?
• What data are available?
• What state variables should be included?
• What processes are needed to represent the dynamic changes in state variables?
• What method should you use to model processes (flows)?
Crop Model Complexities

Potential yield increasing measures:
- CO2
- Radiation
- Temperature
- Crop characteristics
  - Physiology, phenology
  - Canopy architecture

Potential yield protecting measures:
- Water
- Nutrients
  - Nitrogen
  - Phosphorus

Actual yield increasing measures:
- Weeds
- Pests
- Diseases
- Pollutants

Shift Focus

- From:
  - System modeling and simulation methods
- To:
  - Working with dynamic models (with crop examples, but methods apply to other biological and agricultural system models)

This shift means that we take a model that has already been developed and work with it.

In our class, we will focus on model 1) evaluation (comparison with data from the real system), 2) analyzing uncertainty and sensitivity of a model to various factors, and 3) estimating parameters and their uncertainty.

Furthermore, we will not attempt to cover all methods in the book in the remaining time, but instead cover 2 or 3 of the most important methods and concepts for each topic that we cover.
Discrete Time Model Form

\begin{align*}
U_1(t+\Delta t) &= U_1(t) + g_1[U(t),X(t); \theta] \Delta t \\
U_2(t+\Delta t) &= U_2(t) + g_2[U(t),X(t); \theta] \Delta t \\
&\vdots \\
U_S(t+\Delta t) &= U_S(t) + g_S[U(t),X(t); \theta] \Delta t
\end{align*}

What do you see in these equations?
• Euler representation of model with S state variables and thus S equations.
• \( U_i \) are the state variables for \( i = 1, 2, \ldots, S \)
• Vector form: \( U = [U_1(t), U_2(t), \ldots, U_S(t)]^T \)
• Differential equations are the \( g_i \) functions
• What is \( U \) relative to \( U_i \)?
• What is \( X(t) \)? (explanatory variables) What is \( \theta \)? (vector of parameters: \( \theta_1, \theta_2, \ldots, \theta_R \) if there are \( R \) parameters)
• If \( \Delta t = 1 \), equation appears as it does in many models
• Still a continuous model in state variables but referred to as a discrete model in time
Notation Discussion
Simple model, p 4

\[ \Delta TT(j) = \max \left[ \frac{TMIN(j) + TMAX(j)}{2} - T_{base}, 0 \right] \]

\[ \Delta B(j) = RUE(1 - e^{-K \cdot LAI(j)})I(j) \]
\[ = 0 \]

\[ \Delta LAI(j) = \alpha \cdot \Delta TT(j) \cdot LAI(j) \max[LAI_{max} - LAI(j), 0] \]
\[ = 0 \]

\[ TT(j) \leq TT_M \]
\[ TT(j) > TT_M \]

\[ TT(j) \leq TT_L \]
\[ TT(j) > TT_L \]
Dynamic Models Viewed as Response Models or Functions

Take an example model,

\[ U(t+1) = U(t) + k \cdot U(t) \]

(the familiar biological or exponential growth model)

Then, we can solve for \( U(t+2) \):

\[ U(t+2) = U(t+1) + k \cdot U(t+1), \text{ which can be rewritten as:} \]

\[ U(t+2) = [U(t) + kU(t)] + k [U(t) + kU(t)], \text{ or} \]

\[ U(t+2) = U(t) + 2kU(t) + k^2U(t) \]

This can be repeated so that \( U(T) \) can be computed for any \( T>1 \)
Dynamic Models Viewed as Response Models or Functions

Thus,
\[ U(T) = f(X, \theta) \]
Which does NOT have \( U(t) \) on the rhs anymore
This can also be written as:

\[ Y = f(X, \theta) \]

Where \( Y \) is the response.
Examples are fertilizer trials,
(yield as a function of \( N \) applied), etc.
Random Model Dynamic Equations and Response Form

Dynamic Equations:
\[ U_i(t+\Delta t) = U_i(t) + g_i[U(t),X(t);\theta]\Delta t + \eta_i \]

Where \( \eta_i \) is a random variable for rate equation \( i \)

Response form:
\[ Y = f(X,\theta) + \varepsilon \]
Where \( \varepsilon \) is a random variable
Statistical Notions
Appendix

• Random Variable
• Cumulative probability distribution and density functions
• Several random variables
• Expectation, variance, covariance, correlation
• Normal distribution
• Conditional distribution
• Estimators
• Bayesian & Frequentist statistics
How to deal with these random variables in solutions

- Monte Carlo Sampling
  - Random variables of model equations
  - Random variables of responses
  - Random variables associated with individual parameters (not mentioned in Chapter 1)
    - i.e., $\theta_i$ uncertainty could be represented by a mean value and variance if it is normally distributed. Monte Carlo simulation could also be used to sample possible values from its distribution, as in the model and response equations. But, if parameters are not independent, random sampling would need to be done from a multivariate distribution
Monte Carlo Sampling
Discussion

- Other