Hydraulics

- Hydraulics is the branch of sciences that deals with practical applications of water and other fluids. It includes hydrostatics, which deals with fluids at rest and hydrodynamics, which addresses forces exerted by or upon fluids in motion.

Water

- Almost a universal solvent
- Exists on earth in all three phases: liquid, gas, and solid
- Liquid density (62.5 lb/ft$^3$ or 1 g/cm$^3$) that changes very little with temperature
- Most dense at 38.4°F (4°C).
Energy

- Energy is the capacity to do work. Three forms of energy must be considered regarding the hydraulics of irrigation systems:
  - pressure energy,
  - kinetic energy, and
  - potential energy.

Pressure Energy

- Fluids must be contained to possess significant pressure energy.
  \[ h_p = \frac{P}{\gamma} = \frac{P}{\rho g} \quad (12.1) \]
- Water contained in irrigation pipes can have a relatively large amount of pressure energy, while energy in water in open channels is limited to the depth of water in this channel.

Pressure Energy

- Usually, pressure is expressed as force per unit area. The units of pressure in the U.S Customary units are pound per square inch (psi) or lb/ft², while in SI system they are expressed in Pascals (Pa=N/m²), kilo-Pascals (kPa), bars or atmospheres (atm). The most common unit of pressure in the US is psi.
Pressure Energy

- For water in the pressurized pipe, the equivalent depth is the height to which water would rise in a stand pipe due to the pressure in the system. This height is called pressure head which is the height of a column of water that exerts a given pressure and is expressed in units of length (ft or m).

\[
\begin{align*}
V & = 1 \text{ cu ft} \\
A & = 1 \text{ sq ft} \\
A & = 144 \text{ sq in} \\
p & = \frac{F}{A} \\
p & = 62.4 \text{ lbs/sq ft} \\
p & = 0.433 \text{ psi} \\
1 \text{ ft water} & = 0.433 \text{ psi}
\end{align*}
\]

Pressure Energy

- A column of water with the height of 1 ft will exert the pressure of 0.433 psi and pressure of 1 psi will raise a column of water 2.31 ft high.
  - 1 ft $H_2O = 0.433$ psi
  - 1 psi $= 2.31$ ft $H_2O$
The static pressure exerted by the column of water is independent of the size, shape, and length of the vessel or pipeline.

Pressure Energy
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Atmospheric Pressure
- Draw a vacuum
- 1 atmosphere of pressure = 14 psi or 101.3 kPa
Potential Energy

- In an irrigation system the elevation head is a measure of the potential energy in the system. Elevation head is measured in units of length (ft or m). This can be directly compared to the pressure head which can be expressed in the length units. The elevation head can be converted also to pressure units.

Potential Energy

![Diagram of an irrigation system]

Kinetic Energy

- The kinetic energy in an irrigation system is expressed as velocity head.
- It is a measure of the energy required to produce a certain velocity in a fluid.
Kinetic energy is expressed in terms of the velocity head. The velocity head is typically relatively small in comparison to pressure and elevation heads for most pressurized irrigation systems, though is of more substantial importance in open channel irrigation and drainage systems.

\[ h_v = \frac{v^2}{2g} \]  

Note that the expression \( v^2/2g \) has units of length which are consistent with units of pressure head and elevation. This is why the kinetic energy of the fluid is often referred to as velocity head.
**System Head**

- At any point the sum of the elevation, pressure and velocity heads represents the system head at that point.

\[ h_{sys} = h_e + h_p + h_v = y + \frac{p}{\gamma} + \frac{v^2}{2g} \quad (12.3) \]

- The system head graphed relative to the length of the system represents the energy grade line, and indicates the potential that allows fluid to flow. The sum of the pressure and elevation heads is commonly referred to as the hydraulic grade line.

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The valve is closed at point E. Relative to point E, what is the system head at points A, B, C, D and E?

- **A, B, C**: 170 ft.
- **D & E**: 170 ft.

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The valve is closed at point E. What is the pressure at points A, B, C, D and E?

- **A**: 21.6 psi (g)
- **B & C**: 43.3 psi
- **D & E**: 73.6 psi
**Fundamental Flow Equation**
- Simply put the fundamental flow equation states that the average flow rate exiting a control boundary of known cross-sectional area is equal to average velocity of the fluid normal to that cross-sectional area times that area.

\[ Q = \int_A \vec{v} \cdot d\vec{A} \quad (12.4) \]

\[ Q = Av \quad (12.5) \]

**Continuity Equation**
- In general the continuity equation is given by the partial differential equation

\[ \frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{v}) \quad (12.6) \]

- Which expanded is

\[ \frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial x}(\rho v_x) - \frac{\partial}{\partial y}(\rho v_y) - \frac{\partial}{\partial z}(\rho v_z) \quad (12.7) \]

**Continuity Equation**
- Through integration and use of the chain rule the continuity equation can be written in the form of the following

\[ Q_{in} = Q_{out} + \frac{\Delta S}{\Delta t} \quad (12.8) \]

- Combining the fundamental flow equation with the continuity equation gives

\[ A_v v_{in} = A_{out} v_{out} \quad (12.9) \]
Bernoulli Equation

- Bernoulli stated that
  \[ \frac{mv^2}{2} + mgv + Vp = \text{constant} \]  \(12.10\)
- rearranged
  \[ \frac{v^2}{2g} + y + \frac{p}{\rho g} = \text{constant} = h_{sys} \]  \(12.11\)

\[ h_{sys} = h_p + h_v + h_e \]
Modified Bernoulli Equation

- The Bernoulli equation was modified with a lump loss parameter to account for the differences between the measured system head as water flow from one point to another

\[ h_i + h_n + h_{i2} - h_{loss} = h_{f1} + h_{f2} \]  
\[ h_{sys} - h_{loss} = h_{sys} \]  

(12.12)  
(12.13)

- Total head losses the sum of the loss due to friction along the length of the conduit plus the friction due to all the minor elements along the length of the conduit

\[ h_{loss} = h_f - \sum h_m \]  

(13.14)
A_1 = 6 sq in  \quad h_{losses} = 2.4 \text{ ft}  
\begin{align*} 
v_1 &= 2.5 \text{ fps} \\
h_1 &= 0.1 \text{ ft} \\
h_2 &= 0 \text{ ft} \\
h_p &= 23.1 \text{ ft} 
\end{align*}  
A_2 = 3 sq in  
\begin{align*} 
v_2 &= 5 \text{ fps} \\
h_1 &= 0.4 \text{ ft} \\
h_2 &= 0 \text{ ft} \\
h_p &= 20.4 \text{ ft} 
\end{align*}

**Minor Losses I**

- One method for minor loss determination is use the velocity head adjusted minor loss calculation. In general, minor losses are proportional to the velocity head

\[ h_m = k_m \frac{v^2}{2g} \quad (12.15) \]

<table>
<thead>
<tr>
<th>Component</th>
<th>Minor Friction Loss Coefficient, k_m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gate Valve: 100% opened</td>
<td>0.39</td>
</tr>
<tr>
<td>75% opened</td>
<td>1.1</td>
</tr>
<tr>
<td>50% opened</td>
<td>4.8</td>
</tr>
<tr>
<td>25% opened</td>
<td>27</td>
</tr>
<tr>
<td>Globe Valve: opened</td>
<td>10</td>
</tr>
<tr>
<td>Angle Valve: opened</td>
<td>4.3</td>
</tr>
<tr>
<td>Butterfly Valve: opened</td>
<td>12</td>
</tr>
<tr>
<td>Check Valve</td>
<td>4.0</td>
</tr>
<tr>
<td>Coupling</td>
<td>0.10</td>
</tr>
<tr>
<td>90 Elbow</td>
<td>0.35</td>
</tr>
<tr>
<td>Expansion: 25% increase</td>
<td>0.16</td>
</tr>
<tr>
<td>100% increase</td>
<td>0.57</td>
</tr>
<tr>
<td>400% increase</td>
<td>0.92</td>
</tr>
<tr>
<td>Contraction: 20% reduction</td>
<td>0.18</td>
</tr>
<tr>
<td>50% reduction</td>
<td>0.37</td>
</tr>
<tr>
<td>80% reduction</td>
<td>0.49</td>
</tr>
<tr>
<td>Tee: line flow</td>
<td>0.35</td>
</tr>
<tr>
<td>Tee: branch flow</td>
<td>1.25</td>
</tr>
</tbody>
</table>
Darcy-Weissbach Equation

- The Darcy-Weissbach formula is a lumped parameter model that accounts for friction loss along the length of a uniform conduit

\[ h_f = \lambda \left( \frac{1}{D} \right) \frac{v^2}{2g} \]  \hfill (12.16)

Smooth Conduit Flow

- For hydraulically smooth pipes (PVC, PE, Aluminum) the friction factor is given from

\[ \frac{1}{\sqrt{\lambda}} = 2 \log \left( \frac{R_c \sqrt{f}}{2.51} \right) \]  \hfill (12.17)
Rough Conduit Flow

- For hydraulically rough pipes (concrete, culverts) the friction factor is given from

\[
\frac{1}{\sqrt{f}} = -2 \log \left( \frac{1}{3.71 \, D} \right) \tag{12.18}
\]

Colebrook-White Equation

- The Prandtl and the von Karman Equations were combined to solve for flow from the hydraulically rough to the smooth

\[
\frac{1}{\sqrt{f}} = -2 \log \left( \frac{1}{3.71 \, D} + \frac{2.51}{R_s \sqrt{f}} \right) \tag{12.18}
\]

Swamee-Jain Equation

- The Swamee-Jain equation was developed as a non-iterative alternative to the Colebrook-White equation

\[
\frac{1}{\sqrt{f}} = -2 \log \left( \frac{1}{3.71 \, D} + \frac{5.74}{R_s \sqrt{f}} \right) \tag{12.19}
\]
Power Law Formulas

- Through all ranges of flow, velocity can be written in terms of a power law relationship of the hydraulic radius and the hydraulic gradient
  \[ v = k R_0^n S_0^m \] (12.20)
- The hydraulic gradient is defined as
  \[ S_0 = \frac{h_j}{l} \] (12.21)

Power Law Formulas

- Friction loss can be rearranged as
  \[ h_f = k \frac{v^m R_0^n}{l} \] (12.22)
- The hydraulic gradient is defined as
  \[ R_0 = \frac{A}{P} \] (12.23)

Power Law Formulas

- For a pipe or other circular cross-section conduit the hydraulic radius is defined as
  \[ R_0 = \frac{D}{4} \] (12.24)
- The friction loss can then be written in term of the radius
  \[ h_f = \left( k \frac{v^m}{\left( \frac{k}{4^m} \right)^{1/m}} \right) \frac{v^m D^{1/m}}{l} = k' v^m D^{1/m} l \] (12.25)
Power Law Formulas

- The derivative of the friction loss with respect to the derivative of the velocity
  \[ dh_f = \frac{1}{m} k_v^{\frac{1}{m}} D^{-\frac{1}{m}} dv \]  
  (12.26)
- Dividing by the result by the friction loss yields the relative error of friction loss in terms of the relative error of the velocity
  \[ \frac{dh_f}{h_f} = \frac{1}{m} \frac{dv}{v} = \varepsilon_f = \frac{1}{m} \varepsilon_v \]  
  (12.27)

Hazen-Williams Formula

- The basic form of the Hazen-Williams formula is
  \[ v = k_s C R_h^{0.634} S^{-0.54} \]  
  (12.28)
- For English units the coefficient is
  \[ k_s = 0.001^{1.4} \approx 1.318 \]  
  (12.29)
- For metric units the coefficient is
  \[ k_s = 0.001^{0.6} \times 0.3048^{0.37} \approx 0.8493 \]  
  (12.30)

Hazen-Williams Formula

- Friction loss in English units can be written as
  \[ h_f = 4.726 \left( \frac{Q}{C} \right)^{1.852} D^{-4.867} l \]  
  (12.31)
- Friction loss in Metric units can be written as
  \[ h_f = 10.67 \left( \frac{Q}{C} \right)^{1.852} D^{-4.867} l \]  
  (12.32)
Hazen-Williams Formula

- Friction loss in specialized English units can be written as

\[ h_f = 10.45 \left( \frac{Q}{C} \right)^{1.832} D^{-0.867} l \]  \hspace{1cm} (12.33)

Hazen-Williams Formula

- Comparing the Darcy-Weisbach friction factor to the Hazen-Williams formula the following relationship is developed

\[ \lambda \propto \frac{1}{\sqrt{0.1451 D^{0.6867}}} \]  \hspace{1cm} (12.34)
Manning Formula

- The basic form of the Manning formula is
  \[ v = \frac{K}{n} \left( \frac{R_s^n}{S_o} \right)^{\frac{3}{2}} \]  \hspace{1cm} (12.35)
- For English units the coefficient is
  \[ \kappa = 0.3048^{\frac{1}{n}} = 1.486 \]  \hspace{1cm} (12.36)
- For metric units the coefficient is
  \[ \kappa = 1.000 \]  \hspace{1cm} (12.37)

Friction loss in English units can be written as

\[ h_j = 4.662n^{2} Q^{2} D^{-5.333}l \]  \hspace{1cm} (12.38)

Friction loss in Metric units can be written as

\[ h_j = 10.29n^{2} Q^{2} D^{-5.333}l \]  \hspace{1cm} (12.39)

Friction loss in specialized English units can be written as

\[ h_j = 13.18n^{2} Q^{2} D^{-5.333}l \]  \hspace{1cm} (12.40)
Comparing the Darcy-Weisbach friction factor to the Manning formula the following relationship is developed:

\[ \lambda \propto \frac{1}{D^{1.333}} \]  

(12.41)

Another method for minor loss determination is to use the equivalent length method. In this method one uses an equivalent length straight pipe into a prescribed friction loss equation, this method simplifies the overall calculation.

\[ h_{\text{loss}} = 10.45 \left( \frac{Q}{C} \right)^{1.832} D^{-4.867} \left( f + \sum f_{eq} \right) \]  

(12.42)

Minor Losses II

• Another method for minor loss determination is to use the equivalent length method. In this method one uses an equivalent length straight pipe into a prescribed friction loss equation, this method simplifies the overall calculation.
Types of Flow

- Steady flow
- Transient flow

- Uniform flow
- Gradually varied flow
- Rapidly varied flow

Types of Flow

- Pressurized Flow
- Gravity Flow

- Laminar Flow
- Turbulent Flow

- Subcritical
- Critical
- Supercritical