Laterals

- Laterals are branched-network conduits that bring water to the field for application.
- For sprinkler and microirrigation, laterals are either pipes or tubing that are connected to sprinkler nozzle or microirrigation emitter.
- Risers, drop tubes, and spaghetti tubes may be employed, but are not branched.
Mains and Sub-mains

- Mainlines are branched network conduits that supply either laterals directly, or sub-mains from the irrigation water source.
- Sub-mains, when used, represent an intermediate branched network that is supplied by a mainline and supplies laterals.
Manifolds

- Manifolds are valved branched networks.
- Mainlines and sub-mains can be manifolds as well.
- By definition laterals are not manifolds, the lateral would technically begin after the valve body, even if this means the lateral supplies a single nozzle or port.

Design Criteria

- Keller & Bliesner: Lateral inlet pressure should generate an average lateral pressure that is equal to the required sprinkler operating pressure.
- Typically, the average lateral pressure is found 40% down the lateral from the mainline (or submain), provided that the lateral is going down gradient.
- The difference in flow rate should not exceed 10%, from the beginning to the end of any given lateral.
Orientation

- Lateral, mains and submains should run downhill to take advantage of elevation change for pressure head when possible. This also aids flushing and draining the system. Though steep slopes may require the “dreaded pressure regulator”.
- No pressure variation in the lateral can be achieved when the ground slope is equal to the frictional slope, however the frictional slope is nonlinear and flow dependant.

Orientation

- Orienting laterals, mains and sub-mains along contours is typical, and pressure variation will be due to friction losses alone.
- Laterals should not be laid uphill, pressure variation would then come from elevation changes and friction loss. Though there are instances when it becomes cost effective to minimize main and/or sub-main lengths. Remember where to place low pressure drains, for these special cases.

Lateral Design

- Lateral inlet pressure is equal to
  \[ p = \frac{3}{4} p_f + \frac{2}{5} p_r + p_r \]  
  (23.1)
- Therefore the inlet pressure head is found from
  \[ h = \frac{3}{4} h_f + \frac{2}{5} h_r + h_r \]  
  (23.2)
Lateral Design

- Lateral inlet pressure is equal to

\[ x = \frac{2}{5} L \]  

(23.3)

The following relationship is commonly used

\[ h = \bar{h} + \frac{3}{4} h_f + \frac{1}{2} \bar{h}_s + h_r \]  

(23.4)

Though the basis of this equation is questionable, especially for downhill slopes the inlet pressure head equation is for "quick 'n' dirty" calculations, or "back of the envelope" designs. Actual inlet pressures should be determined from network analysis.
Uniform Flow, Uniform Spacing
- The uniform spacing between laterals is given by
  \[ l = \frac{L}{n} \] \hspace{1cm} (23.5)
- For this first example, the pipe (tube) diameter will be the same throughout
  \[ D_i = D \] \hspace{1cm} (23.6)

Uniform Flow, Uniform Spacing
- It is also assumed that the flow rate is the same out of each emitter (this is the major assumption), so that the total flow is given by
  \[ Q = nq \] \hspace{1cm} (23.7)
- And the flow through each portion of pipe preceding a given emitter is
  \[ Q = iq \] \hspace{1cm} (23.8)

Uniform Flow, Uniform Spacing
- The friction loss through each portion of pipe preceding a given emitter is
  \[ h_{ij} = k Q^n_{ij} D^{-2\alpha-\beta} \] \hspace{1cm} (23.9)
- This can be rewritten as
  \[ h_{ij} = k \left( \frac{Q}{n} \right)^{n} D^{-2\alpha-\beta} \frac{L}{n} \] \hspace{1cm} (23.10)
**Uniform Flow, Uniform Spacing**

- The friction loss along the length of the entire lateral is given by

\[
h_{\ell} = \sum_{i=1}^{n} k Q^\alpha D^{2\alpha - \beta} l
\]  

(23.11)

- This can be rewritten as

\[
h_{\ell} = k Q^\alpha D^{2\alpha - \beta} L \left[ \frac{1}{n^{\alpha+1}} \sum_{i=1}^{n} i^\alpha \right]
\]  

(23.12)

---

**Uniform Flow, Uniform Spacing**

- Christensen found that the modifying coefficient to be close to

\[
\frac{1}{n^{\alpha+1}} \sum_{i=1}^{n} i^\alpha \approx F_c = \frac{1}{\alpha + 1} + \frac{1}{2n} + \frac{\sqrt{\alpha + 1}}{2n^2}
\]  

(23.13)

- The modifying coefficient can be found using the Euler-Maclaurin Summation

\[
\sum_{i=1}^{n} f(i) \approx \int_1^n f(x) \, dx + \frac{1}{2} (f(1) + f(n)) + \frac{1}{12} f''(\frac{1}{2}) + \frac{1}{24} f''(\frac{3}{2}) + \frac{1}{720} f^{(4)}(\frac{1}{2}) + \cdots
\]  

(23.14)

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**Uniform Flow, Uniform Spacing**

- The modifying coefficient can be found from

\[
\frac{1}{n^{\alpha+1}} \sum_{i=1}^{n} i^\alpha = F = \frac{1}{\alpha + 1} + \frac{1}{2n} + \frac{\alpha}{12n^2} + \frac{\alpha^2}{720n^4} + \cdots
\]  

(23.15)
Uniform Flow with Extra Outflow, Uniform Spacing

- It is still assumed that the flow rate is the same out of each emitter (again this is the major assumption) and there is an additional outflow, so that the total flow is given by

\[ Q = nq + Q_o \]  (23.16)

- And the ratio of the outflow relative flow through the emitters is

\[ r = \frac{Q_o}{nq} \]  (23.17)

Uniform Flow with Extra Outflow, Uniform Spacing

- And the flow through each portion of pipe preceding a given emitter is

\[ Q_i = iq + Q_o \]  (23.18)

- This can be rewritten as

\[ Q_i = q(i + nr) \]  (23.19)
**Uniform Flow with Extra Outflow, Uniform Spacing**

- Using the same methodology and the Euler-Maclaurin Summation, the modifying coefficient can be found for the extra outflow condition

\[
F_r = \frac{1}{n^{R_{r+1}}(1+r)^\alpha} \left\{ \frac{[n(1+r)+1]^{\alpha+1}}{\alpha+1} - \frac{[nr]^{\alpha+1}}{\alpha+1} - \frac{[n(1+r)+1]^2}{2} + \frac{[nr]^2}{2} - \frac{[n(1+r)+1]^3}{12} + \frac{[nr]^3}{12} \right\}
\]

\[(23.20)\]

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**Uniform Flow, Uniform Spacing with Shortened Inlet Length**

- The uniform spacing between laterals is given by

\[
l = \frac{L}{n + x - 1}
\]

\[(23.21)\]

- Or the total length of the lateral is given by

\[
L = (n + x - 1)l
\]

\[(23.22)\]
Uniform Flow with Extra Outflow, Uniform Spacing

- Using the same methodology as before, the modifying coefficient can be found for the shortened inlet length condition

\[ F_i = \frac{nF + x - 1}{n + x - 1} \]  (23.23)

- or

\[ F_i = \frac{nF_i + x - 1}{n + x - 1} \]  (23.24)

Lateral Constraints and Sizing

- Try to avoid steep slopes

\[ -h_e > 0.30\bar{h} \]  (23.25)

- The change in pressure along a lateral (mainline, or sub main) should not be greater than 20% (when the exponent, \( x \), in the emitter equation is 0.5), based on

\[ \frac{\Delta Q}{Q} = \frac{\Delta p}{p} = 10\% \]  (23.26)

Lateral Constraints and Sizing

- Since there is a 20% variation from the average pressure, then

\[ 0.20\bar{h} = h_f + h_e \]  (23.27)

- Then the allowable friction loss is

\[ h_f = 0.20\bar{h} - h_e \]  (23.28)

- Thus the lateral diameter is given from

\[ D = \left( F - \frac{kQ^x L}{0.20\bar{h} - h_e} \right)^{0.25} \]  (23.29)
Generally the Hazen-Williams formula is used, which gives
\[ D = 1.62\left(\frac{Q}{C}\right)^{0.809}\left(\frac{FL}{0.20\overline{h} - h}\right)^{0.2055} \]  

Sometimes it is cost effective to taper the laterals (using continuously smaller and smaller diameters down the laterals length).

For two different sized pipes, size the upstream leg of the pipe using the preceding procedure with the \( F_r \) (or \( F_x(F_r) \)) where the additional outflow is the total flow to the downstream leg. Remember that only part of the allowable percentage (and elevation effects) can be used for this leg.

Size the downstream leg of the lateral using the above preceding procedure with the simple \( F \) where the inflow is the additional outflow of the upstream lateral. Remember that only part of the allowable percentage (and elevation effects) can be used for this leg, and should be the remainder of the total allowable from the upstream leg.

More than two different diameters is rare, but follows a similar procedure.
Lateral Constraints and Sizing

- Sub-mains, and mainlines are sized based on the same procedure, the constant outflows in this case do not go to “perfect” nozzles, they flow into “perfect” laterals.
- This procedure sizes pipes adequately for most purposes, but does not quantify actual flow rate or pressures very well.

Network Analysis

\[ h = \left( \frac{Q}{k'} \right)^\gamma + h_i + h_w + h_m + h_r \]  \hspace{1cm} (23.31)

\[ h_1 = \left( \frac{q_1}{k_{1'}} \right)^\gamma + h_i \]  \hspace{1cm} (23.32)

\[ h_1 = \left( \frac{Q_1}{k_{1'}} \right)^\gamma + h_i \]  \hspace{1cm} (23.33)
Network Analysis

\[ h_2 = \left( \frac{q_2}{k_{32}} \right)^{\frac{1}{2}} + h_r \]  
(23.34)

\[ h_3 = \left( \frac{Q_3 - Q_r}{k_{32}} \right)^{\frac{1}{2}} + h_r \]  
(23.35)

\[ h_1 = \left( \frac{q_1}{k'_{21}} \right)^{\frac{1}{2}} + h_r \]  
(23.36)

Network Analysis

\[ h_5 = \left( \frac{Q_5 - Q_r}{k'_{53}} \right)^{\frac{1}{2}} + h_r \]  
(23.37)

\[ h_7 = kQ^aD^{2a-\beta}L = JQ^a \]  
(23.38)

\[ h_1 = h_2 - JQ^a_2 - h_r \]  
(23.39)

Network Analysis

\[ h_2 = h_1 - JQ^a_1 - h_r \]  
(23.40)

\[ h_3 = h - JQ^a_3 - h_r \]  
(23.41)

\[ f_1 = h_2 - JQ^a_2 - h_r - h_1 = 0 \]

\[ = \left( \frac{Q_2 - Q_r}{k'_{21}} \right)^{\frac{1}{2}} - JQ^a_2 - h_r - \left( \frac{Q_1}{k'_{1}} \right)^{\frac{1}{2}} \]  
(23.42)
Network Analysis

\[ f_3 = \left( \frac{Q_3 - Q_2}{k_3} \right)^2 - JQ_x' = 0 \quad (23.43) \]
\[ f_2 = h - JQ_x' - h' = 0 \quad (23.44) \]

In order to achieve the correct solution for flows and pressure heads, the values for \( f_1, f_2, \) and \( f_3 \) must be zero. To that end the first order Newton-Rhapson procedure will be employed, which utilizes the Taylor series expansion:

\[ f(x + \Delta x) = f(x) + \frac{df}{dx}(x)\Delta x + \frac{1}{2!}\frac{d^2f}{dx^2}(x)\Delta x^2 + \ldots \quad (23.45) \]

The first order Newton-Rhapson procedure uses the truncated series

\[ f(x + \Delta x) = f(x) + \frac{df}{dx}(x)\Delta x \quad (23.46) \]
Network Analysis

- Next use a little matrix algebra/"madness" with three differential equations

\[
\begin{bmatrix}
\frac{df_1}{dQ_1} & \frac{df_1}{dQ_2} & \frac{df_1}{dQ_3} \\
\frac{df_2}{dQ_1} & \frac{df_2}{dQ_2} & \frac{df_2}{dQ_3} \\
\frac{df_3}{dQ_1} & \frac{df_3}{dQ_2} & \frac{df_3}{dQ_3} \\
\end{bmatrix}
\begin{bmatrix}
\Delta Q_1 \\
\Delta Q_2 \\
\Delta Q_3 \\
\end{bmatrix}
= 
\begin{bmatrix}
\frac{f_1}{f_2} \\
\frac{f_2}{f_3} \\
\frac{f_3}{f_1} \\
\end{bmatrix}
\]

(23.47)

Network Analysis

- Simplifying, by assuming the Hazen-Williams friction loss

\[\alpha = 1.852\]  \hspace{1cm} (23.48)

- And for three sprinklers they will have the same nozzles

\[K = k_1' = k_2' = k_1', \quad x = \frac{1}{2}\]  \hspace{1cm} (23.49)

Network Analysis

\[
\frac{df_1}{dQ_1} = \frac{2}{K}(Q_2 - Q_1) - 1.852JQ_1^{k_{882}}
\]  \hspace{1cm} (23.50)

\[
\frac{df_2}{dQ_1} = \frac{2}{K}(Q_3 - Q_1)
\]  \hspace{1cm} (23.51)

\[
\frac{df_3}{dQ_1} = 0
\]  \hspace{1cm} (23.52)

\[
\frac{df_2}{dQ_1} = \frac{2}{K}(Q_2 - Q_1)
\]  \hspace{1cm} (23.53)
Network Analysis

\[
\frac{df_2}{dQ_1} = -\frac{2}{K} (Q_1 - Q_2) - 1.852JQ_1^{0.852} \quad (23.54)
\]

\[
\frac{df_2}{dQ_2} = \frac{2}{K} (Q_1 - Q_2) \quad (23.55)
\]

\[
\frac{df_2}{dQ_1} = 0 \quad (23.56)
\]

\[
\frac{df_2}{dQ_2} = \frac{2}{K} (Q_1 - Q_2) \quad (23.57)
\]

Network Analysis

\[
\frac{df_1}{dQ_1} = -\frac{2}{K} (Q_1 - Q_2) - 1.852JQ_1^{0.852} \quad (23.58)
\]

\[
\frac{df_1}{dQ_2} = \frac{2}{K} (Q_1 - Q_2) \quad (23.59)
\]

\[
\frac{df_1}{dQ_1} = 0 \quad (23.60)
\]

\[
\frac{df_1}{dQ_2} = \frac{2}{K} (Q_1 - Q_2) \quad (23.61)
\]

\[
h_1 = \frac{(Q_1)}{K} + h_r \quad (23.33)
\]

\[
h_2 = \left( \frac{Q_1}{K} \right) + h_r \quad (23.39)
\]

\[
h_3 = \left( \frac{Q_1}{K} \right) + h_r \quad (23.40)
\]

\[
h_4 = \left( \frac{Q_1}{K} \right) + h_r \quad (23.40)
\]
**Center Pivots**

- Unlike all other laterals, the center pivots lateral or "arm" requires that there be different nozzle sizes down the length of the pivot, or varying distances between same size nozzles. Though the latter is rarely used.
- Nozzle size must vary because the effective area each nozzle covers increases down the lateral.

\[ q_i = \pi \left[ \frac{R + \left( \frac{3 - 2i}{2} \right)}{2} \right]^2 - \left[ \frac{R + \left( \frac{1 - 2i}{2} \right)}{2} \right]^2 \] (23.59)
Deviation from Design Flow Rate, gpm

Distance from Pivot, ft

20 Nozzle Span X 0.37 Nozzle Spacing
5.125” ID
Senniger LDN Nozzles, 5/64” to 3/8”
Instantaneous Application Depth 20 in/hr
Pivot Pressure 15 psi