Model Evaluation of Top and Root Accumulation with Time by Corn

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ABSTRACT

The expanded growth model provides a mathematical relationship for accumulation of biomass with time for plant tops (above-ground portion). In this article a hyperbolic relationship is shown to provide reasonable coupling between root and top accumulation with time for corn (Zea mays L.). These results suggest that the rate limiting step is biomass accumulation in the tops by photosynthesis, and that root-to-top coupling can be treated as quasi-equilibrium.

INTRODUCTION

Overman[1] introduced the expanded growth simulation model to describe dry matter accumulation with time by plants. The model is derived from

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a linear-exponential intrinsic growth function and a Gaussian environmental function. Overman and Scholtz\cite{2} then applied the model to data from several field studies with corn in the Southeastern United States. In the present analysis we extend the analysis to include root dry matter accumulation along with that in the tops for corn.

**MODEL DESCRIPTION**

The expanded growth model for accumulation of dry matter with time by plant tops (above-ground portion), $Y_t$, is described by \cite{1}:

\[ Y_t = AQ \]  \hspace{1cm} (1)

where $A =$ yield factor, and the growth quantifier, $Q$, is defined by

\[ Q = \exp\left(\sqrt{2\sigma c}x_1\right) \]

\[ \times \left\{ (1 - kx_1)[\text{erf} x - \text{erf} x_1] - \frac{k}{\sqrt{\pi}}[\exp(-x^2) - \exp(-x_1^2)] \right\} \]  \hspace{1cm} (2)

Dimensionless time, $x$, is defined by

\[ x = \frac{t - \mu}{\sqrt{2\sigma}} + \frac{\sqrt{2\sigma c}}{2} \]  \hspace{1cm} (3)

where $t =$ calendar time since Jan 1, wk; $t_i =$ time to initiation of significant growth, wk; $x_1 =$ value of $x$ corresponding to $t_i$; $\mu =$ time to the mean of the yield distribution, wk; $\sigma =$ standard deviation of the yield distribution, wk; $c =$ aging factor in the intrinsic growth function, wk$^{-1}$; $k =$ curvature factor in the intrinsic growth function. The error function is defined by

\[ \text{erf} x = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-u^2)du \]  \hspace{1cm} (4)

This model describes accumulation of biomass in the tops of the plants by photosynthesis. Based on examination of data we assume that root biomass, $Y_r$, can be related to top biomass by the phase equation

\[ Y_r = \frac{Y_{rm}Y_t}{K_t + Y_t} \]  \hspace{1cm} (5)

where $Y_{rm} =$ potential maximum root biomass, and $K_t =$ coefficient of
response for top biomass. Equation (5) can be rearranged to the linear form

\[
\frac{Y_t}{Y_r} = \frac{K_t}{Y_{rm}} + \frac{1}{Y_{rm}} Y_t
\]

(6)

DATA ANALYSIS

Data for this analysis are taken from a field study by Foth\[3\] with corn grown on Conover loam (fine-loamy, mixed, active, mesic Udollic Endoaqualfs) at East Lansing, MI. Row by drill spacing was 107 cm \times 20.3 cm. Results are listed in Table 1 and shown in Fig. 1, where biomass is reported as Mg ha\(^{-1}\). It may be noted that dry matter of both tops and roots increased with elapsed time, whereas the root/top ratio decreased with time.

Model simulation requires estimates of model parameters. Based on results from Overman and Scholtz\[2\] and Fig. 1 we assume the following values:

\[ t_i = 27.0 \text{ wk}, \quad \mu = 30.0 \text{ wk}, \quad \sqrt{2}\sigma = 8.0 \text{ wk}, \quad c = 0.20 \text{ wk}^{-1}, \quad k = 5. \]

This leads to the equations

\[
x = \frac{t - 30.0}{8.0} + 0.8 = \frac{t - 23.6}{8.0} \Rightarrow x_i = 0.425
\]

(7)

\[
Q = 1.974\{ -1.125[\text{erf}(x) - 0.452] - 2.821[\exp(-x^2) - 0.835]\}
\]

(8)

Table 1. Accumulation of top and root biomass for corn grown at East Lansing, MI (1960).\(^a\)

<table>
<thead>
<tr>
<th>Date</th>
<th>Time (wk)</th>
<th>Tops (Mg ha(^{-1}))</th>
<th>Roots (Mg ha(^{-1}))</th>
<th>Tops/roots</th>
<th>Roots/tops</th>
</tr>
</thead>
<tbody>
<tr>
<td>06/27/60</td>
<td>25.6</td>
<td>0.051</td>
<td>0.025</td>
<td>2.0</td>
<td>0.49</td>
</tr>
<tr>
<td>07/11/60</td>
<td>27.6</td>
<td>0.53</td>
<td>0.201</td>
<td>2.6</td>
<td>0.38</td>
</tr>
<tr>
<td>07/15/60</td>
<td>28.1</td>
<td>1.22</td>
<td>0.383</td>
<td>3.2</td>
<td>0.31</td>
</tr>
<tr>
<td>07/21/60</td>
<td>29.0</td>
<td>2.06</td>
<td>0.565</td>
<td>3.6</td>
<td>0.28</td>
</tr>
<tr>
<td>07/28/60</td>
<td>30.0</td>
<td>4.05</td>
<td>0.791</td>
<td>5.1</td>
<td>0.20</td>
</tr>
<tr>
<td>08/10/60</td>
<td>31.9</td>
<td>7.62</td>
<td>0.815</td>
<td>9.3</td>
<td>0.11</td>
</tr>
<tr>
<td>08/23/60</td>
<td>33.7</td>
<td>8.72</td>
<td>1.14</td>
<td>7.7</td>
<td>0.13</td>
</tr>
<tr>
<td>09/12/60</td>
<td>36.6</td>
<td>12.65</td>
<td>1.18</td>
<td>10.7</td>
<td>0.094</td>
</tr>
</tbody>
</table>

\(^a\) Data adapted from Foth.\[3\]
Figure 1. Accumulation of top and root biomass with time by corn grown at East Lansing, MI (1960). Data adapted from Foth.[3] Curves drawn from Eqs. (7) through (11).
Top and Root Accumulation with Time

Estimates are given in Table 2 for \( Q \) vs. \( t \) for the growth period of the study, where the erf is evaluated from math tables.\(^{[4]}\) The yield function is calibrated from

\[
Y_t = \left( \frac{13.5}{3.433} \right) Q = 3.93Q
\]

by assuming maximum top yield of 13.5 Mg ha\(^{-1}\). Values of \( Y_t \) in Table 2 are calculated from Eq. (9). The next step is to estimate the parameters in the phase relationship. The phase plot is shown in Fig. 2. Equation (6) is evaluated by linear regression to obtain

\[
\frac{Y_t}{Y_r} = 2.28 + 0.703Y_t \quad r = 0.976
\]

with a correlation coefficient of 0.976. This leads immediately to the hyperbolic phase equation

\[
Y_r = \frac{1.42Y_t}{3.24 + Y_t}
\]

which results in the curve in Fig. 2. Values of \( Y_r \) in Table 2 are calculated from

<table>
<thead>
<tr>
<th>( t ) (wk)</th>
<th>( x )</th>
<th>( \text{erf}_x )</th>
<th>( \exp (-x^2) )</th>
<th>( Q ) (Mg ha(^{-1}))</th>
<th>( Y_t ) (Mg ha(^{-1}))</th>
<th>( Y_r ) (Mg ha(^{-1}))</th>
<th>( Y_r/Y_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>0.425</td>
<td>0.452</td>
<td>0.835</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.438</td>
</tr>
<tr>
<td>28</td>
<td>0.550</td>
<td>0.563</td>
<td>0.739</td>
<td>0.288</td>
<td>1.13</td>
<td>0.37</td>
<td>0.325</td>
</tr>
<tr>
<td>29</td>
<td>0.675</td>
<td>0.660</td>
<td>0.634</td>
<td>0.657</td>
<td>2.58</td>
<td>0.63</td>
<td>0.244</td>
</tr>
<tr>
<td>30</td>
<td>0.800</td>
<td>0.742</td>
<td>0.527</td>
<td>1.071</td>
<td>4.21</td>
<td>0.80</td>
<td>0.191</td>
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<tr>
<td>31</td>
<td>0.925</td>
<td>0.810</td>
<td>0.425</td>
<td>1.488</td>
<td>5.85</td>
<td>0.91</td>
<td>0.156</td>
</tr>
<tr>
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<td>0.862</td>
<td>0.332</td>
<td>1.890</td>
<td>7.43</td>
<td>0.99</td>
<td>0.133</td>
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<tr>
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<td>0.903</td>
<td>0.251</td>
<td>2.250</td>
<td>8.84</td>
<td>1.04</td>
<td>0.118</td>
</tr>
<tr>
<td>34</td>
<td>1.300</td>
<td>0.934</td>
<td>0.184</td>
<td>2.555</td>
<td>10.0</td>
<td>1.07</td>
<td>0.107</td>
</tr>
<tr>
<td>35</td>
<td>1.425</td>
<td>0.956</td>
<td>0.131</td>
<td>2.801</td>
<td>11.0</td>
<td>1.10</td>
<td>0.100</td>
</tr>
<tr>
<td>36</td>
<td>1.550</td>
<td>0.972</td>
<td>0.0905</td>
<td>2.991</td>
<td>11.8</td>
<td>1.11</td>
<td>0.094</td>
</tr>
<tr>
<td>37</td>
<td>1.675</td>
<td>0.9821</td>
<td>0.0605</td>
<td>3.136</td>
<td>12.3</td>
<td>1.12</td>
<td>0.091</td>
</tr>
<tr>
<td>38</td>
<td>1.800</td>
<td>0.9989</td>
<td>0.0392</td>
<td>3.239</td>
<td>12.7</td>
<td>1.13</td>
<td>0.089</td>
</tr>
<tr>
<td>39</td>
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<td>0.9995</td>
<td>0.0246</td>
<td>3.310</td>
<td>13.0</td>
<td>1.14</td>
<td>0.087</td>
</tr>
<tr>
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<td>2.050</td>
<td>0.9961</td>
<td>0.0150</td>
<td>3.358</td>
<td>13.2</td>
<td>1.14</td>
<td>0.086</td>
</tr>
<tr>
<td>( \infty )</td>
<td>( \infty )</td>
<td>1</td>
<td>0</td>
<td>3.433</td>
<td>13.5</td>
<td>1.15</td>
<td>0.085</td>
</tr>
</tbody>
</table>
Figure 2. Phase plot of root biomass vs. top biomass for corn grown at East Lansing, MI (1960). Data adapted from Foth.\cite{3} Line drawn from Eq. (10); curve from Eq. (11).

Eq. (11), and are shown in Fig. 1. Corresponding values of the root/top ratio are listed in Table 2 and plotted in Fig. 1.

DISCUSSION

The expanded growth model provides reasonable simulation of dry matter accumulation by the tops for corn in this study, as observed for other studies in Florida, North Carolina, and South Carolina.\cite{2} It appears from the present analysis that a phase relationship provides reasonable coupling between root and top biomass. It is concluded that the rate limiting step is biomass accumulation in the tops by photosynthesis, and that coupling between roots
and tops can be treated as a quasi-equilibrium process. Likewise, it appears from the analysis by Overman and Scholtz\cite{2} that plant N accumulation by corn tops can be treated as quasi-equilibrium with photosynthesis as the rate limiting step.

REFERENCES